

CBCS/ SEMESTER SYSTEM

(w.e.f. 2020-21 Admitted Batch)

B.A./B.Sc. MATHEMATICS

COURSE-II, THREE DIMENSIONAL ANALYTICAL SOLID GEOMETRY

Time: 3Hrs

Max.Marks:75 M

SECTION - A

Answer any FIVE questions. Each question carries FIVE marks 5 X 5 M=25 M

1. Find the equation of the plane through the point $(-1,3,2)$ and perpendicular to the planes $x+2y+2z=5$ and $3x+3y+2z=8$.
2. Find the bisecting plane of the acute angle between the planes $3x-2y-6z+2=0$, $-2x+y-2z-2=0$.
3. Find the image of the point $(2,-1,3)$ in the plane $3x-2y+z=9$.
4. Show that the lines $2x + -4 = 0 = y + 2z$ and $x + 3z - 4 = 0, 2x + 5z - 8 = 0$ are coplanar.
5. A variable plane passes through a fixed point (a, b, c) . It meets the axes in A,B,C. Show that the centre of the sphere OABC lies on $ax^{-1}+by^{-1}+cz^{-1}=2$.
6. Show that the plane $2x-2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$ and find the point of contact.
7. Find the equation to the cone which passes through the three coordinate axes and the lines $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and $\frac{x}{2} = \frac{y}{1} = \frac{z}{1}$.
8. Find the equation of the enveloping cone of the sphere $x^2 + y^2 + z^2 + 2x - 2y = 2$ with its vertex at $(1, 1, 1)$.

SECTION - B

Answer ALL the questions. Each question carries TEN marks. 5 X 10 M = 50 M

9(a) A plane meets the coordinate axes in A, B, C. If the centroid of $\triangle ABC$ is

(a,b,c) , show that the equation of the plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$.

(OR)

(b) A variable plane is at a constant distance p from the origin and meets the axes in A,B,C. Show that the locus of the centroid of the tetrahedron OABC is

$$x^2+y^2+z^2=16p^2.$$

10(a) Find the shortest distance between the lines

$$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}; \quad \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}.$$

(OR)

(b) Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}; \frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. Also find their point of intersection and the plane containing the lines.

11 (a) Show that the two circles $x^2+y^2+z^2-y+2z=0$, $x-y+z=2$;

$x^2+y^2+z^2+x-3y+z-5=0$, $2x-y+4z-1=0$ lie on the same sphere and find its equation.

(OR)

(b) Find the equation of the sphere which touches the plane $3x+2y-z+2=0$ at $(1,-2,1)$ and cuts orthogonally the sphere $x^2+y^2+z^2-4x+6y+4=0$.

12 (a) Find the limiting points of the coaxial system of spheres $x^2+y^2+z^2-8x+2y-2z+32=0$, $x^2+y^2+z^2-7x+z+23=0$.

(OR)

(b) Find the equation to the cone with vertex is the origin and whose base curve is $x^2+y^2+z^2+2ux+d=0$.

13 (a) Prove that the equation $\sqrt{fx} \pm \sqrt{gy} \pm \sqrt{hz} = 0$ represents a cone that touches the coordinate planes and find its reciprocal cone.

(OR)

(b) Find the equation of the sphere $x^2+y^2+z^2-2x+4y-1=0$ having its generators parallel to the line $x=y=z$.